Development and Application of Methodology for Assisted History Matching
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Introduction

History matching is an inverse problem that consists of finding the best combination of a set of reservoir parameters such the simulation model reproduces observed data. The objective is to improve reservoir models by incorporating the observed (or dynamic) data, such as pressure, seismic and production data, into the characterization process, in order to obtain reliable production forecasting.

The process involves the selection of the parameters, systematic changes in these parameters, flow simulation of the resulting models, evaluation of the objective function, that measure the quality of the matching, that is, a large amount of tasks that difficult the manual process. Assisted history matching allows to automate part of the process and can improve the results once more evaluation and iterations are possible.

Several assisted history matching methodologies have been proposed in the literature. Most of the works can be classified in two major categories. The first one, normally called local optimization, uses gradient-based optimization methods, that requires calculating derivatives (Arenas, 2001; Brun, 2001; Wu, 2001; Rodrigues, 2005). Into the class of gradient-based methods, Rodrigues (2005) mentions three subclasses: Gauss-Newton method, that requires full sensitive matrix; conjugate gradient, for which products of sensitivity matrix and its transpose are needed and quasi-Newton methods, for which is necessary the gradient of the objective function. According Rodrigues, the implementation of derivative calculation techniques in existing complex simulator is a difficult task.

The second category, also called global optimization methods, uses algorithms that do not require derivative computation (Romero, 2000; Schulze-Riegert, 2002). These algorithms do not require any gradient information and use only the objective function value.

Schulze-Riegert (2002) used evolutionary algorithm, that belongs to the class of direct optimization method, for history matching of complex fields. According to the authors, evolutionary algorithms are capable of searching beyond local optima and have the potential to identify configurations in the search space of model parameters that generate acceptable solutions.

Some works combine the two methodologies, as for example, Gomez (2001) and Mantica (2002). Comparing local and global optimization, some authors (Mantica, 2002, for example) argue about the slow convergence of the latter. It is important to observe, however, that convergence rate is strongly dependent on the nature of each case. For example, a synthetic case with known solution, without non-linearities or discontinuities of the space solution, both methods certainly works with fast convergence.

However, for complex case, with high non-linearities or discontinuities of the space solutions, as result of complex interactions between parameters and objective function, both methods present problem. The presence of local minima represents a challenge for any optimization methods.
The objective of this paper is to present a methodology for assisted history matching based on direct optimization methods, which falls to the global optimization category.

**Methodology**

The methodology developed in this work uses a direct search method in the optimization process.

Direct search method consists of the discretization of the matching parameters. The dimension of the solution space depends on the number of parameter and on the discretization level of the parameters. The search method is performed by successive exploratory and linear search. An initial point pertaining to the solution space is selected (point 1 in Fig. 1) and the neighboring points are evaluated. Each evaluated point implies in a flux simulation. The point of smaller objective function defines the direction of the linear search. The point of the linear search with smaller objective function is used to a new exploratory search and so on, until a minimum is founded.

In this paper, the integration of three improvements is proposed: independent objective function, heuristics and linear diagonal search.

**Independent Objective Function.** This consists of to separate the domain in sub solution spaces in which an optimization process is performed simultaneously and independently. Each sub solution space is composed by a set of regional parameters. The schematic representation is showed in Fig. 1, where appears an example of two solution spaces with two parameters each one. It is also possible to use different number of parameters for different space solutions.

The objective function is calculated according to the least-square method, defined as follows:

$$\text{IOF}_j = \sum_{i=1}^{p} (d_{i,j}^{\text{obs}} - d_{i,j}^{\text{sim}})^2,$$

where $j$ is the number of independent objective function (IOF), $p$ is the number observed data, $d_{i,j}^{\text{obs}}$ is the observed data, such as water rate, water cut, well bottom hole pressure, for example, and $d_{i,j}^{\text{sim}}$ is the simulated data.

**Heuristics.** This part of the methodology corresponds to the choice of several initial points, by sampling the solution space (For example, points H in Fig. 1b). These points are evaluated and that correspondent to the smaller objective function is selected as the initial guess to the algorithm. Each sub solution space can be sampled in different way, that is, different number of points in different positions can be evaluated. It is important to note that choosing a starting point with smaller objective function (among several points) do not guarantees, but increases the probability of a global minimum be reached.

**Diagonal Linear Search.** In the methodology proposed by Leitão and Schiozer (1999) and Schiozer (1999), the direction of the linear search is always defined by the point of smaller objective function among the points of the exploratory search. In this paper, the direction of the linear search depends on the relationship between the points with smaller objective function. Consider that points 2a and 2b in the example in Fig. 2 have the smallest objective function. Defining F12a as the percent reduction of the objective function from point 1 to point 2a, and F12b as the percent reduction of the objective function from point 1 to point 2b, the relationship between F12a and F12b defines one of the possible directions illustrated in the Fig. 2. For example, if $(F12a/F12b) ≅ 1$, the direction of the linear search is defined by the blue arrow. If $1<(F12a/F12b)<2$, the direction of the linear search is defined by the green arrow.

**Applications**

The methodology presented in this paper was applied to two cases. The first example application (Case 1) is a synthetic reservoir and the second is a real field (Case 2)

**Case 1.** This case is a heterogeneous reservoir represented through a corner point grid composed by 90 elements in x direction, 34 elements in the y direction and 10 layers. The reservoir is characterized by 5 regions, delimited by faults, forming 5 different facies. The reservoir is drained by 5 horizontal producer wells and 5 horizontal water injector wells. This fine grid was simulated and the output was considered as the observed data (production history).

A coarser model (to be adjusted) was generated through an upscaling process. The upscaling factors were 3 in the x direction, 2 in the y direction and 2 in the z direction. Equivalent porosity was found by arithmetic mean and the vertical and horizontal permeability were upscaled using the method proposed by Maschiodo and Schiozer (2003). Each set of 12 elements of the fine grid was merged in one element with volume correspondent to the 12 elements, such the reservoir volume represented by the coarse grid is equal to the volume represented by the fine grid.

In Fig. 3a is the three-dimensional map of the horizontal permeability and Fig. 3b shows the first layer (top layer) with the 5 producer wells. The injector wells are completed in the same areal positions, in the fifth layer.

To match the water cut of the 5 producer wells, three history matching procedures were used. The first one was using the improvements proposed in this paper (Match 1), the second consisted in dividing the process in 5 steps (Match 2) and the third consisted in dividing the process in two steps (Match 3). The problem corresponded to 20 matching parameters. Horizontal and vertical permeability, the exponents and end points of the water relative permeability of the 5 regions. The second step was the matching of the others 10 permeability parameters. In the first and second step of the Match 3, the objective function was composed through the combination all wells water cut.
In the three procedures, 16 multipliers (15 intervals) between 0.25 and 3.0 were used for horizontal and vertical permeability. The exponents and the end points for water relative permeability curves (Corey model) varied between 1 and 5 and between 0.15 and 0.9, respectively, both with 15 intervals.

**Case 2.** The application 2 is a real offshore field drained by 37 producer vertical wells and 13 water injector vertical wells. It is a developed field, with 22 years of production history and more than 15 years of water injection. Fig. 4 shows three-dimensional vertical permeability for Case 2.

Four misadjusted wells (W15, W24, W35 and W26) were selected for water rate history matching. Horizontal and vertical permeability were used as regional parameters for each well. For each parameter, 30 multipliers were used. The extreme multipliers depended on the well, but followed an overall variation between 0.1 and 3.

Again, three matching procedure were used, as described in the Case 1. For Case 2, however, the third procedure (Match 3) differs slightly from Case 1. Once the number of parameters was not so large (8), a unique step was used.

**Results and Discussions**

**Case 1.** In Figs. 5-9 is shown the results for final history match of well water cut using the three procedures for Case 1. As can be seen, overall quality of the match is very good. Despite some disagreement in wells PROD1 and PROD4, mainly in the breakthrough period, the solutions are very similar for the three procedures.

Table 1 summarizes the number of simulation for each matching procedure. Match 1, which uses the proposed methodology, required only 73 simulations, while Match 2 required 255 simulations and Match 3 required 574. Through the analysis of this numbers and observing the quality of the matching, it is clear the advantage of the Match 1. Another aspect is the comparison between the number of simulation of Match 2 and Match 3. Taking into account the same quality of solutions, the division of the process in steps with less parameters was more interesting.

In Figs. 10-14 is presented the evolution of the objective function (normalized with respect to the initial model) for Match 1 and 2. Firstly, the high reduction is evident for all wells and a high convergence rate can also be observed. With respect to the number of simulations seem in these plots, some aspects can be discussed. For example, for Match 1 the overall process took 73 simulations, however, some processes (related to PROD3 and PROD4) finished earlier.

**Case 2.** In Figs. 15, 16, 17 and 18 are presented the results of the final history match for wells W15, W24, W35 and W36, respectively. The three solutions (Match 1, Match 2 and Match 3) shows overall good match. Table 2 shows the number of simulations for Case 2. Again, its clear the advantage of the proposed method (Match 1), that used 38 simulations, while Match 2 used 115 simulations and Match 3 used 286 simulations.

The evolution of the objective function for Case 2 (Match 1 and Match 2) is presented in Figs. 19-22. Although the reduction level is smaller when compared with Case 1, that was expected because Case 2 is a real case, the reduction was significant. The smallest reduction was observed in well W36 (46 % for Match 1 and 68 % for Match 2), due to two main reasons: first, observed data are very scattered and it is more difficult for the simulator to capture this variability and, second, simulated result related to the initial model is, in average, not very distant from observed data.

**Conclusions**

The greater the number of matching parameters, the greater the computational effort in the history matching process. This paper presented a methodology, using direct search optimization technique, in order to accelerate the history matching process. Three methods were integrated: independent objective function, multiple starting points and linear search. The results from two applications, a synthetic and a real field, showed the benefits of the methodology. In the two cases, the reduction of the number of simulations was significant, maintaining the quality of the match.

**References**


Acknowledgments
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Tables and Figures

Table 1 - Number of simulation for three history matching procedures (Case 1)

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Table 2 - Number of simulation for three history matching procedures (Case 2)

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Figure 1 - Schematic representation of the direct search algorithm with two independent space solutions (a and b)

Figure 2 - Linear search schemes

Figure 3 - (a) Three-dimensional horizontal permeability (mD) and (b) horizontal permeability of the top layer showing producer wells (Case 1)
Figure 4 - Three-dimensional vertical permeability (mD) for Case 2

Figure 5 - History matching of well PROD1 (Case 1)

Figure 6 - History matching of well PROD2 (Case 1)

Figure 7 - History matching of well PROD3 (Case 1)

Figure 8 - History matching of well PROD4 (Case 1)

Figure 9 - History matching of well PROD5 (Case 1)
Figure 10 - Evolution of the objective function for PROD1 (Case 1)

Figure 11 - Evolution of the objective function for PROD2 (Case 1)

Figure 12 - Evolution of the objective function for PROD3 (Case 1)

Figure 13 - Evolution of the objective function for PROD4 (Case 1)

Figure 14 - Evolution of the objective function for PROD5 (Case 1)

Figure 15 - History matching of well W15 (Case 2)
Figure 16 - History matching of well W24 (Case 2)

Figure 17 - History matching of well W35 (Case 2)

Figure 18 - History matching of well W36 (Case 2)

Figure 19 - Evolution of the objective function for W24 (Case 2)

Figure 20 - Evolution of the objective function for W36 (Case 2)

Figure 21 - Evolution of the objective function for W15 (Case 2)

Figure 22 - Evolution of the objective function for W35 (Case 2)