An Implicit Treatment of Upscaling in Numerical Reservoir Simulation
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Abstract
In this paper, a multiscale computational model for multiphase flow that implicitly treats upscaling without using pseudo-functions is proposed. It overcomes some practical difficulties related to the use of the traditional pseudo-curves by executing the upscaling and solution processes in just one step and taking into account changes in the numerical model in an adaptive manner.

Introduction
Numerical simulation of petroleum reservoirs is associated with intensive use of computational resources. Advances in petroleum reservoir descriptions have provided an amount of data that can not be used directly in flow simulations. Geostatistical techniques are able to generate descriptions of heterogeneous reservoirs with great detail, in a very fine scale. This detailed geological information must be incorporated into a coarser model during multiphase fluid flow simulation by using some upscaling technique. In numerical simulation processes, equations are discretized and the flow domain is divided into blocks with associated rock properties. This step requires that the geological description be transferred from the flow properties model to the reservoir simulation. Due to computational limitations it is not possible to run multiphase flow simulations at such a scale and it is necessary to perform upscaling of properties and solve the problem on a coarser grid.

In single-phase flow, the most important parameter to upscale is absolute permeability, methods for which are well established. But when multiphase flow occurs, it is also necessary to adjust the phase flow through the connections of the coarse grid. In such cases, the most widely used upscaling technique employs pseudo relative permeabilities. The Kyte and Berry method (Kyte and Berry (1975)) is the most common approach applied to calculate pseudo-curves. This procedure requires two steps: (1) generation of pseudo-curves for each block of the coarser grid, and (2) simulation of the model considering such functions. In addition to these generation steps there are limitations associated to these pseudo-curves that restrict their use in a more general way.

The methodology of the proposed procedure uses parameters generated from numerical flow simulation in some regions of the domain to create an equivalence between the description and the simulation scales. By solving a sequence of local problems on the more refined scale, it is possible to achieve good agreement between a coarse grid and a fine grid, without expensive computations on a fine grid model of the whole reservoir. This procedure does not use multiphase pseudo-function concepts and avoids the computational cost of solving the fine grid.

The examples presented here consider two-dimensional, two-phase flow (oil and water) and the Black-Oil formulation. Results of flow simulation considering homogeneous and heterogeneous porous media are presented and are also used to compare this approach with commercial upscaling software.

Upscaling Techniques Using Pseudo-Functions
Employment of pseudo-functions is the traditional manner to perform upscaling. It consists of replacing original saturation-dependent functions on a certain scale by fictitious ones, which represents the same physical process in a coarser solution mesh.

Numerical and analytical simplified models can be used to construct pseudo-functions. Analytical methods are suitable when simplified assumptions are valid. Coats et al. (1971) derived pseudo-functions for vertical equilibrium conditions based on gravity-capillary equilibrium. Hearn (1971) extended this procedure to non-communicating layers.

Space and time-dependent functions were introduced by Jacks et al. (1973) to overcome the rate limitations of the vertical pseudos. To obtain these dynamic functions for each coarse block it is necessary to run numerical models in a section of the reservoir. Jacks et al. (1973) proposed a methodology based on simulation of 2D cross-sections generating a set of pseudo relative permeability curves.
representing each column and ran the final model in a two-dimensional areal model.

Emanuel and Cook (1974) extended the pseudo relative permeability functions concepts to fit the vertical performance of individual wells. By using a similar technique, they proposed well pseudos for well completions in the coarse grid. Based on examples of different completion schemes, the authors showed that the proposed procedure works for almost all cases presented.

The most common method to calculate dynamic pseudo-curves was proposed by Kyte and Berry (1975). These authors developed a method based on Darcy’s law to calculate pseudo-functions. It is considered an extension of Jacks et al. (1973) and includes pseudo capillary pressure curves. Despite being popular and used as a reference it doesn’t give good results in strongly heterogeneous media, and some inconsistencies, such as negative or infinite values of relative permeability, can occur.

Based on the Kyte and Berry (1975) approach, Lasseret et al. (1986) presented a multiscale upscaling methodology suitable for heterogeneous reservoirs. Using some particular reservoir permeability distributions, these authors showed how reservoir heterogeneities at small, medium and large scales influence the ultimate recovery and how they affect the multiphase behavior. The proposed pseudo-function generation process begins at the laboratory scale, and the next largest scale can be achieved by replacing effective properties determined at the previous scale.

Starley (1988) presented a procedure based on material balance to derive pseudo relative permeability curves with application to two-dimensional problems. The method is similar to Jacks et al. (1973) and focuses on the matching of fluid fluxes between interfaces of a reference fine grid and a coarse grid areal model. This method includes a dispersion control scheme to offset numerical dispersion and works only for the specific displacement process for which it was derived.

Kossack et al. (1989) described a multi-step scale up process to consider several scales of heterogeneities in two-phase, incompressible displacements. Three geological descriptions of heterogeneities (homogeneous, layered and random) and various groups of fluid flow regimes were tested. Extensive numerical experiments were run to verify the effects of different flow regimes on the pseudo-function curves in the three geological descriptions. As in Lasseret (1986), the pseudo-functions did not consider grid blocks adjacent to wells and used Kyte and Berry (1975) method in its derivation.

Stone (1991) was the first one to use the average total mobility to avoid calculating phase potential on the coarser grid, as Kyte and Berry (1975) require. He introduced a fractional flow formulation instead of calculating the flow terms by Darcy’s law. This method can be applicable even to non-communicating layers. Hewett and Berhens (1991) and Beir (1992) proposed other methodologies based on averaged total mobility.

Alabert and Corre (1991) presented an approach to deal with 3D models of geological heterogeneity generated by geostatistical conditional simulation techniques, covering three-phase flow. Effective flow parameters were obtained from simulation in a certain direction, to each coarse block and for all simulation time steps. Averaged relative permeability curves and capillary pressures are thus constructed for each pair of fluids and for three directions.

Pickup and Sorbie (1994) showed that heterogeneous media require a tensorial effective relative permeability approach. According to these authors pseudo-functions must be derived from flow in different directions to account for the effects of directional flow.

Guérillot and Verdière (1995) and Verdière and Thomas (1996) developed an upscaling scheme based on two grids: a coarse one used by the pressure equation and a fine one used to define the saturation distribution. This procedure, named the Dual Mesh Method, treats discretization in space and time and performs an adaptive upscaling of the parameters for the pressure equation.

A review of the use of pseudo relative permeabilities for upscaling can be found in the work of Barker and Thibeau (1996). Barker and Dupouy (1996) present a comparison among five different methods of dynamic pseudo relative permeability.

In spite of the high number of methods proposed, there is no general and automatic procedure to perform upscaling and all methodologies based on pseudo-functions have drawbacks and restrictions associated with the character of these functions. Some properties of pseudo-functions, such as non-unique, rate and initial conditions dependence, make the process unreliable and it is necessary to search for an alternative approach for upscaling.

A Multiscale Computational Model

This paper presents a new upscaling technique to be used in the modeling of flow in porous media in a grid which is coarser when compared with the scale where the properties were described. The main objective of the proposed methodology is to obtain solutions on a coarser grid with fine mesh accuracy, without the limitations of pseudo-functions and without fine grid computational costs. With this computational model, upscaling becomes an adaptive process, since all changes in the model are automatically taken into account during the coarse grid simulation.

The basic idea of the proposed methodology is to use the parameters provided by numerical simulation in certain refined regions, called subdomains, to create an equivalence between the simulation grid and the fine grid where the petrophysical properties are described. During simulation, all scales shown on Fig. 1 are present in the computational model. To obtain the adjustment of flow terms at the coarse grid model, some information must be exchanged between coarse and fine
scales. Also the equations of the coarse model are altered to incorporate the upscaling process.

**Algorithm.** The flowchart shown on Fig. 2 describes the main processes carried out at each time step using the multiscale computational method. The following items detail each of these processes.

1. **Initialization of the coarse grid**
   This step includes a sequence of procedures to join blocks of the fine grid to construct the subdomains and all blocks of the coarse grid. Once geometrically defined, the next step is to perform the upscaling of absolute permeability and to associate parameters to each block.

2. **Solution of a sequence of local problems**
   This is the first step of the time-step solution. The main objective here is to get information about phase flow distribution on the fine mesh by solving the problem in all subdomains using Dirichlet pressure boundary conditions. Considering that only an approximation of this distribution is desired, a large tolerance can be used in the iterative solution method. Dirichlet conditions at an auxiliary boundary marked with * on Fig. 3 can be established either as:
   - Constant pressure and equal to the next subdomain, or
   - Interpolated pressure where heterogeneities are considered.

   These boundary condition calculations were based on the model proposed by Hermite and Guerilot (1993).

3. **Matching flow terms of the coarse grid**
   At this step, a representative saturation of each subdomain is obtained based on the saturation distributions performed at Step II. A representative saturation value, named \( S_p^* \), is established based on the saturation distribution of the fine grid. The desired goal here is to fit phase transmissibilities between coarse grid blocks in such a manner that they can approximate the flow at correspondent interfaces at fine grid.

   The phase transmissibility is defined by:
   \[
   T_p = f_g f_{sp} f_p \quad \text{........................................... (1)}
   \]

   Where \( f_g \) represents geometric factors, \( f_{sp} \) represents phase saturation dependent factors, and \( f_p \) represents pressure dependent factors.

   In pseudo-function methods, the flow matching between blocks is carried out by phase saturation-dependent factors. Likewise, in this formulation, this term will be used to perform such adjustments inside the simulator during the time-step calculations. A representative saturation value \( (S_p^*) \) is obtained for each subdomain and the original relation \( (k_p \text{ versus } S_p) \) is applied.

   The use of \( S_p^* \) is also an important aspect of the multiscale method and different alternatives were used in this research. The options tested can be observed on Fig. 4 where two alternatives are shown (gray and dark gray). The value of \( S_p^* \) is calculated based on the saturation of the fine grid blocks weighted by the phase mobilities.

   The representative phase saturation \( S_p^* \) is obtained before the coarse grid solution, and it is used only to evaluate coarse interblocks transmissibilities. These saturation values are obtained by a sequence of local problems solved at subdomains with Dirichlet boundary conditions. These refined domains are able to describe the front saturation response with better accuracy than a coarse grid.

   An important aspect related to \( S_p^* \) is its restricted use to obtain saturation-dependent terms during the calculation of transmissibilities. The computation of the primary variables at coarse grid is independent of this parameter.

4. **Matching source terms of the coarse grid**
   This step is aimed at adjusting the source terms by modifying well index and phase transmissibilities through connections between blocks and wells. Well indices are calculated using absolute permeabilities of the fine grid and the phase transmissibilities are calculated based on \( S_p^* \). Both permeability and \( S_p^* \) are based on the fine grid block, where the well is perforated.

5. **Solution on the coarse grid**
   At this step, the problem is solved using a fully implicit formulation, with the phase flow being corrected according to the transmissibilities calculated in Steps II & IV. The coarse grid model is fully implicit, except for the use of a constant \( f_{sp} \) term during the time step.

6. **Distribution of coarse grid connection flow over each subdomain boundary**
   The phase flow at coarse block interfaces obtained in Step V is distributed over the external boundary of each subdomain. This distribution is weighted by the upstream transmissibilities at each fine grid interface.

7. **Matching coarse grid and subdomains**
   Subdomains are reinitialized and solved again with Neumann boundary conditions to update the fine grid using the results of adjusted coarse grid interblocks flow. This step permits the evolution of the two scales in a consistent way. The distribution of the flux to the fine grid is proportional to the term \( T_p \Delta P_p \).

   Different from Step II, the iterative solution process at each subdomain at this step has to converge to a solution within a specified tolerance. This step does not begin with the solution of step II (which is just to obtain \( S_p^* \)). It is here where the full time step is calculated.

**Discussion.** Several boundary conditions were used during this research. At the **Step II**, the best solution was obtained by using Dirichlet boundary conditions (Fig. 3) based on the pressure of the neighboring block using the adjacent subdomain. Further research is being carried out to investigate if a different boundary condition is necessary in other examples.
The only objective of using $S_p^*$ with the original relative permeability curves is to correct the term $T_p$. The best options can be observed on Fig. 4 where two alternatives are represented (gray and dark gray), but further research is necessary to investigate the efficiency of the method under different conditions (heterogeneity, phase mobility, importance of gravity, etc.).

More details about the algorithm can be found in Guedes (1998).

**Numerical Results**

One of the objectives of this research was to develop a procedure that could be easily incorporated into conventional numerical simulator schemes. For this reason, a finite difference numerical, two-dimensional and two-phase, Black-Oil model was developed and a solver based on the ORTHOMIN procedure and a preconditioner, constructed by incomplete LU-factorization (Brand (1992)) was implemented. The proposed methodology was incorporated in this conventional computational code and some tests were carried out for validation. Gravity effects were included, but capillary pressure was neglected.

Guedes (1998) presented a complete analysis of the performance of the numerical model developed and the influence of different problems on the results of the multiscale simulator. These problems cover different fluid characteristics (densities and compressibilities), different descriptions of porous media and distinct forms of displacements of oil by water.

Two kinds of patterns are considered in the examples: cross-sections and one-quarter of five spot. In both situations, a water injector well and a producer well are represented. In all examples, fluid parameters are the same as Aziz and Odeh (1981) and pressure conditions are imposed in the producer well as a form to evaluate the well flow rate. The relative permeability curves considered are of the Corey type with

$$k_{rw} = S_{w,m} \alpha \cdot k_{ro} = (1 - S_{w,m})^{\alpha}$$  \hspace{1cm} (2)

$$S_{w,m} = \frac{S_w - S_{w,i}}{1 - S_{o,r} - S_{w,i}}$$  \hspace{1cm} (3)

Four cases are shown, named ZEUS, ARES, ODIN, and THOR. The first two, ZEUS and ARES, compare oil fractional flow ($f_o$) with dimensionless time ($t_o$), namely the number of pore volumes of water injected, among three procedures: global fine grid simulation (FG), coarse model simulation without using pseudo-functions (CG), and the proposed multiscale simulator (SCALE). These results refer to $S_{o,i} = S_{o,r} = 0.2$ and $n = 1.5$. For the heterogeneous cross-section represented in ODIN case, the comparison among these three procedures is presented comparing oil rates at the producer well. Finally, for the THOR case a comparison of oil and water flow rates at the producer well is presented between the proposed model, SCALE, and the commercial software PSEUDO. For this last comparison $S_{o,i} = 0.12$, $S_{o,r} = 0.3$ and $n = 1$. PSEUDO uses the Kye and Berry (1975) functions to construct pseudo-functions to blocks, and the Emanuel and Cook (1974) approach to calculate well functions.

**ZEUS Case.** This first example shows a one-quarter of a five-spot pattern. The fine mesh is comprised of 144 squared blocks of 1000 ft while 16 blocks comprise the coarse grid. The permeability distribution shown on Fig. 5 is such that the fine and coarse grids are heterogeneous but the subdomains are homogeneous, therefore no permeability upscaling is necessary.

In this example, the producer is located at block (2,2,1) with prescribed bottom hole pressure of 4000 psia while the water injector well is placed at (11,11,1), with a constant water injection rate (15000 bbl/d). Results of CG, FG and SCALE are presented on Fig. 6. These curves show a good performance of SCALE, when compared with that of the FG.

**ARES Case.** This example shows a homogeneous cross-section represented on Fig. 7. The main purpose of this example is to investigate the behavior of SCALE when different relations of oil and water densities are considered. The fine mesh is comprised of 12x1x6 blocks and the coarse grid of 4x1x1, representing an 18-time upscaling. The dimensions of fine grid blocks are 300 ft x 50 ft x 50 ft. In this example, the perforation of the producer well is located at block (2,1,1) with prescribed bottom hole pressure of 4000 psia while the water injector well is placed at (11,1,5), with a constant water injection rate (1000 bbl/d).

Results of CG, FG and SCALE are presented on Figs. 8 and 9 showing $f_o$ versus $t_o$, for oil-to-water density ratio greater and lower than 1, respectively. We verify a good agreement between FG and SCALE for both situations.

**ODIN Case.** This example shows a heterogeneous cross-section. The permeability distribution is presented on Fig. 10. The objective is to investigate the behavior of SCALE during the presence of heterogeneities such that the preferential flow path coincides with gravitational force effects. The dimensions of the fine grid blocks are 400 ft x 50 ft x 50 ft and the fine and coarse meshes are comprised of 12x1x3 and 4x1x1 blocks, respectively. The producer is located at (1,1,1); the water injector well at (12,1,2) and the operational conditions prescribed are the same for the ARES case.

A comparison among CG, FG and SCALE in terms of oil rate at producer well is shown on Fig. 11. Again, these results represent a very good performance of SCALE when compared with the numerical solution of fine grid.

**THOR Case.** The last case represents an important feature of permeability distribution that can be found in realistic reservoir studies: the presence of channeling in heterogeneous porous media. It represents a special difficulty for upscaling
procedures, given that a channel, in a fine description, can disappear after absolute permeability upscaling. This example has the objective of demonstrating the ability of the proposed procedure to capture the effects of the channel on the saturation distribution during the subdomain solution step and to transfer it to the coarse grid. The permeability distribution is shown on Fig. 12, the geometric parameters are the same ZEUS case and the operational conditions prescribed are: bottom hole pressure of 4500 psia at the producer well and constant water injection rate (10000 bbl/d).

The purpose of presenting results on Fig. 13 is to show the effects of adjusting the flow terms related to wells. This adjustment permits obtaining a good match between FG and SCALE in terms of oil production even before water breakthrough.

On Fig. 14, a comparison between PSEUDO and SCALE is presented. In this example a modification of the Kyte and Berry method, named the Pore Volume Weighted Pseudos, was applied to obtain the PSEUDO response. The PSEUDO results were obtained with the consideration of directional relative permeabilities, and setting in the numerical simulator ECLIPSE the ability to consider values of relative permeabilities greater than 1 and less than zero. Otherwise the results would be much different from those presented on Fig. 14.

The comparison of FG, SCALE and CG is shown on Fig. 15. It can be observed that even for very heterogeneous problems, the multiscale technique is appropriate. It is exactly in such cases that the differences between FG and CG should be significantly greater. However, this must be tested with other examples.

Discussion

The treatment of subdomains during the local-problem solving process has an important difference compared to that given by domain decomposition methods. In the proposed procedure, no convergence is performed among subdomains. They are treated as independent structures and, for this reason, this methodology does not have the convergence problems reported by Nacul (1991). This is a relevant advantage of this procedure.

There are other advantages of the computational model associated to the unified process and computational costs. The methodology does not depend on well positions, rates and initial conditions, as in pseudo-function techniques. All changes in the configuration of the model are automatically incorporated in the solution process. This important characteristic permits to incorporate changes without the regeneration of pseudo-functions.

The objective of the multiscale method is to obtain similar results as close as possible to the results of a fine grid without an excessive computational cost. Sometimes it is impossible to incorporate the details of geostatistical lengthscale in a flow simulation model and the problem can be solved only in a low-resolution mesh.

An important issue related to the new computational model are the computational costs. These costs can be analyzed from a theoretical point of view. Considering that the predominant cost of a reservoir simulator is due to the solution of linear system of equations, the costs can be compared in terms of the number of operations required during this solution step, per iteration. On a conjugate gradient type method the number of operations is \( N \), where \( N \) is the number of unknowns and \( m \) is the average number of elements by row on a sparse matrix.

Consider a two-dimensional problem with \( ij \) blocks at a fine grid and \( k^2 \) at a coarse grid, where \( j \) and \( J \) are related to the smallest dimension. At the fine grid, the number of equations is \( 2ij \) and the band size is \( 2j \). The number of operations (NO) per iterations at fine grid is

\[
NO_{FG} = 8ij(j + 1)................................. (4)
\]

For a multiscale procedure, the number of operations can be expressed by:

\[
NO_{MS} = 8IJ(J + 1) + 2IJ(8kref^2(kref + 1))........ (5)
\]

where \( kref \) is the refinement ratio between fine and coarse scales. Fig. 16 shows the ratio \( NO_{FG}/NO_{MS} \) versus the dimension \( n \) of a subdomain for four fine grids consisting of 144, 400, 3600 and 10000 blocks. It can be observed that the multiscale model is applicable to large problems, as well as in cases where the subdomains contain a great number of blocks (high \( n \)). However, in order to affirm this, it is necessary to test additional problems. In addition, to make a fair comparison of run times, we must have 2 different simulators with clean codes developed specifically with this objective because the coarse grid used here has many options which would not be necessary if a multiscale procedure was not used.

In summary, it is necessary to simulate larger problems to test the reduction in computational costs and the possible loss of accuracy. More than two scales may be necessary for great reductions from coarse to fine grid.

It is also important to mention that the quality of the solution of these two treatments (globally fine and multiscale with refinement) are different. The second procedure has an overhead of computational costs, which are not present in the first. Except for this overhead, when compared with traditional simulators, the multiscale scheme carries out less operations than the complete solutions of the fine grid model.

Another important point related to the optimization of this procedure is the solution of subdomains. So far, all examples have been solved using all subdomains and in every timestep. Future research is being directed aimed at the possibility of eliminating Steps I and VII for some subdomains using various selection criteria. The implementation of the dynamic selection process and parallel computations of subdomains are expected to represent substantial savings of total computational time.

Future research and additional implementations are
necessary to test problems including the presence of gas, 3D, larger models, different degrees of refinement, local refinement, parallel computing etc.

Summary and Conclusion
This work presents a new computational model that incorporates the upscaling step during numerical simulation of multiphase flow in petroleum reservoirs. This procedure avoids the use of pseudo-functions in an explicit manner, as is usually done. The main conclusions are:

1. The approach has the ability to overcome the limitations of pseudo-functions, since all changes are taken into account during coarse grid simulation in an adaptive manner.
2. The multiscale computational model proposed is able to produce simulation results from the coarse grid model with a resolution equivalent to those from the fine grid. The examples demonstrate a good performance of the SCALE simulator to calculate water breakthrough. Moreover, the proposed model has the ability to take into account heterogeneous media, as showed in the THOR example.
3. A theoretical analysis of the computational costs of this new computational model shows that this scheme is more suitable for large problems and small subdomains. However, new examples must be tested to affirm this in practice.

Nomenclature
\( f \) = fractional flow
\( l \) = number of blocks in \( x \) direction for the coarse grid
\( i \) = number of blocks in \( x \) direction for the fine grid
\( J \) = number of blocks in \( y \) direction for the coarse grid
\( j \) = number of blocks in \( y \) direction for the fine grid
\( k_r \) = relative permeability
\( k_{ref} \) = refinement ratio
\( m \) = average number of elements by row on a sparse matrix
\( N \) = number of unknowns
\( n \) = number of blocks of the subdomain
\( NO \) = number of operations per iteration
\( Q \) = flow rate, bbld [m³/d]
\( S \) = saturation, fraction
\( T \) = phase transmissibility
\( t \) = time
\( x, y, z \) = Cartesian coordinates

Subscripts
\( D \) = dimensionless
\( FG \) = fine grid
\( g \) = geometric
\( m \) = normalized
\( MS \) = multiscale
\( o \) = oil
\( p \) = pressure
\( Sp \) = phase saturation
\( w \) = water

Superscripts
\( e \) = representative of a subdomain
\( n \) = exponent of Corey relations

References

**SI Metric Conversion Factors**

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**Fig. 1 - Simulation scales.**

- a) Fine grid
- b) Coarse grid
- c) Subdomain

**Fig. 2 - Flowchart of multiscale computational model.**

**Fig. 3 - Definition of subdomain $ij$ and adjacent subdomains.**

**Fig. 4 - Representation of fine grid blocks (gray) used to calculate $Sp^*$ considering the flux from block 1 to block 2.**
Fig. 5 - ZEUS Case - Distribution of absolute permeabilities.

Fig. 6 - ZEUS Case - Comparison of oil fractional flow among CG, FG and SCALE.

Fig. 7 - ARES Case. Distribution of absolute permeabilities.

Fig. 8 - ARES Case - Comparison of oil fractional flow among CG, FG and SCALE for oil density greater than water density.

Fig. 9 - ARES Case - Comparison of oil fractional flow among CG, FG and SCALE for oil density lower than water density.

Fig. 10 - ODIN Case - Distribution of absolute permeabilities.
Fig. 11 - ODIN Case - Comparison of the oil rate at the producer well among CG, FG and SCALE as a function of time.

Fig. 12 - THOR Case - Distribution of absolute permeabilities.

Fig. 13 - THOR Case - Comparison of the oil rate at the producer well among FG, SCALE and PSEUDO as a function of time.

Fig. 14 - THOR Case - Comparison of the water rate at the producer well among FG, SCALE and PSEUDO as a function of time.

Fig. 15 - THOR Case - Comparison of the water rate at the producer well among FG, SCALE and CG as a function of time.

Fig. 16 - Ratio of number of operations carried out using an iterative method for solving the linear system of equations, during the solution in a multiscale and fine grid.