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A Simple Procedure to Develop Analytical Curves of IPR from Reservoir Simulators with Application in Production Optimization
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ABSTRACT
This paper describes a simple procedure, based in the work of Vogel and Wiggins, to develop IPR curves at any stage of depletion or at any time, for solution gas-drive reservoirs, for two or three phase flow, from results of a Black-Oil reservoir simulator.

In many applications, the use of numerical simulators to represent reservoir behavior would require an excessive computational time. In such a case, one alternative is to use analytical Inflow Performance Relationships (IPR).

One application of IPR is a production optimization method implemented in this work to choose the best value of production parameters such as tubing diameter and choke size. In this procedure, only discrete values are considered and a economic factor is used to optimize an objective function which is the present value of cumulative oil production.

This procedure is compared with the results obtained from simultaneous simulation of reservoir and production facilities which requires much greater computational time.

This work shows: (1) a simple automatic procedure to generate IPR curves from reservoir simulators results, (2) an optimization procedure where only available values of each production parameter are considered; and (3) that dynamic IPR curves that vary with depletion can be used to represent reservoirs in optimization procedures. Problems related to the development of analytical IPR curves are also discussed.

INTRODUCTION
The estimation of individual performance of oil wells can be used to determine, for instance, an optimum method of production, adequate design of artificial lift, success design stimulation, and treatments and forecast of production performance. In many of these cases, the utilization of numerical simulators results in a very high time consumption while IPR curves can be utilized to represent reservoir performance with low computation effort.

Gilbert developed curves which related flow rate and pressure. He was the first who called them IPR curves.

Weller developed one method to calculate depletion performance in solution-gas drive reservoirs applicable for all saturation conditions considering steady state flow and variable gas-oil ratio.

Vogel presented an empirical method to estimate the pressure-production behavior of oil wells producing from solution-gas drive reservoirs based on reservoir simulator results. He used Weller method to calculate IPR curves with a variety of PVT properties and relative permeability data.

These methods considered only two-phase flow (oil and gas). Brown presented a solution developed in Petrobras to calculate three-phase flow IPR curves. This correlation uses a combination of Vogel type equation for the oil IPR and a constant productivity index (PI) for the water IPR. Oil and water fractions were held constant for all flowing bottom hole pressures.

Wiggins, Russell and Jennings developed analytical IPR curves based on the physical nature of the multiphase flow system. They extended these ideas from two to three-phase flow systems.

In this paper, it is described a simple procedure, based in the works of Vogel and Wiggins, to develop IPR curves at any stage of depletion for solution gas-drive reservoirs using results obtained from a Black-Oil simulator.

IPR curves are used here to optimize production parameters such as tubing diameter and choke sizes. Most of the published work about production optimization, for instance, Carroll, developed non-linear techniques to optimize production performance using some continuous variables. In this work, only discrete values are considered to optimize the objective function which is the present value of cumulative oil production. Therefore, this procedure can be used to optimize several parameters simultaneously with low computation effort.

MATHEMATICAL FORMULATION
The development of the mathematical model is based on the following assumptions: (1) homogeneous limited reservoir; (2) reservoir are initially above bubble point pressure; (3) radial flow, (4) Darcy’s law for multiphase flow applies; (5) gravity and capillary effects are neglected; (6) isothermal conditions; (7) no gas solubility in water; and (8) fully penetrating wellbore.
With these assumptions, Wiggins developed the following equations, which describe the analytical IPR for oil and water phases:

\[
\frac{q_j}{q_{j,\text{max}}} = 1 + C_1 \frac{p_w}{p_r} + C_2 \left( \frac{p_w}{p_r} \right)^2 + C_3 \left( \frac{p_w}{p_r} \right)^3 + C_4 \left( \frac{p_w}{p_r} \right)^4
\]

(1)

where the coefficients are defined as:

\[
C_1 = \left( \frac{k_{ij}}{\mu_i \phi_j} \right)_{j=0} + \left( \frac{k_{ij}}{\mu_i \phi_j} \right)_{j=1} + \frac{1}{2} \left( \frac{k_{ij}}{\mu_i \phi_j} \right)_{j=0}
\]

(2)

\[
C_2 = \left( \frac{k_{ij}}{\mu_i \phi_j} \right)_{j=0} + \left( \frac{k_{ij}}{\mu_i \phi_j} \right)_{j=1} + \frac{1}{4} \left( \frac{k_{ij}}{\mu_i \phi_j} \right)_{j=0}
\]

(3)

\[
C_3 = -\left( \frac{1}{6} \left( \frac{k_{ij}}{\mu_i \phi_j} \right)_{j=0} + \frac{1}{6} \left( \frac{k_{ij}}{\mu_i \phi_j} \right)_{j=1} \right)
\]

(4)

\[
C_4 = \frac{1}{24} \left( \frac{k_{ij}}{\mu_i \phi_j} \right)_{j=0}
\]

(5)

and

\[
D = \left( \frac{k_{ij}}{\mu_i \phi_j} \right)_{j=0} + \left( \frac{k_{ij}}{\mu_i \phi_j} \right)_{j=1} + \frac{1}{6} \left( \frac{k_{ij}}{\mu_i \phi_j} \right)_{j=0}
\]

(6)

and the adimensional variable \( \Pi \) is defined as:

\[
\Pi = \frac{\Delta p}{p_e} = \frac{p_t - p}{p_f}
\]

With Eq. 1, a analytical IPR can be utilized to describe the reservoir if one can estimate the mobility function and its derivatives with respect to pressure.

These coefficients are calculated by running a reservoir simulator once to estimate the mobility function at each reservoir depletion stage. This procedure intends to represent the reservoir behavior through these coefficients at each depletion stage. This is possible because the mobility function is a strong function of depletion (figure a) and does not vary significantly with production flow rate (figure b). More details are given by Cadena. For three-phase flow, mobility function is also influenced by the initial water saturation.

**MODEL APPLICATION**

Figure c shows the schematic calculation procedure which is utilized in this work. An important assumption of this procedure is that if some of the assumptions made in the last section are not valid, the error would be “absorbed” by the coefficients calculation which are obtained from reservoir simulator results.

Two cases are shown: Case 1 which considers two-phase flow and Case 2 which considers three-phase flow.

**Case 1: Two-phase flow (oil and gas)**

Data for Case 1 is described in table a. Well locations are in the extremes of the reservoir in a Cartesian system (49 x 49 blocks). A radial reservoir could be used but a rectangular reservoir was used with Cartesian grid to show a condition different than that specified in the assumptions in the last section.

Figure d present the mobility function profiles of the oil phase for two stages of depletion. The resulting IPR curves, which are shown in figure e, are respectively:

\[
\frac{q_o}{q_{o,\text{avg}}} = 1 - 0.033 \left( \frac{p_w}{p_r} \right) - 0.47 \left( \frac{p_w}{p_r} \right)^2 + 0.12 \left( \frac{p_w}{p_r} \right)^3 - 0.26 \left( \frac{p_w}{p_r} \right)^4
\]

(8)

\[
\frac{q_w}{q_{w,\text{avg}}} = 1 - 0.44 \left( \frac{p_w}{p_r} \right) - 0.26 \left( \frac{p_w}{p_r} \right)^2 - 0.31 \left( \frac{p_w}{p_r} \right)^3 + 0.09 \left( \frac{p_w}{p_r} \right)^4
\]

(9)

**Case 2: Three-phase flow**

Data for Case 2 is described in table b. Considering two different stages of depletion (1% and 8%), analytical IPR curves are developed from mobility functions for each phase and at each stage of depletion.

Figure f shows the mobility function at 1% depletion. It can be observed that the mobility functions are more complex. Therefore (1) higher order polynomials can be used to represent these curves and (2) a greater value of \( \Pi \) must be achieved (at least 0.9 in the cases studied here). Otherwise, wrong IPR curves are obtained.

To achieve greater value of \( \Pi \), a more refined grid can be used. For instance, in this case, the reservoir is divided in a Cartesian grid (56 x 56 blocks) and the well block is refined. More details are given by Cadena.

The resulting IPR curves are respectively:

\[
\frac{N_p}{N_e} = 1\%:
\]

- Oil:

\[
\frac{q_o}{q_{o,\text{avg}}} = 1 - 0.33 \left( \frac{p_w}{p_r} \right) + 0.062 \left( \frac{p_w}{p_r} \right)^2 - 4.58 \left( \frac{p_w}{p_r} \right)^3
\]

\[
+ 5.78 \left( \frac{p_w}{p_r} \right)^4 - 2.4 \left( \frac{p_w}{p_r} \right)^5
\]

(10)

- Water:

\[
\frac{q_w}{q_{w,\text{avg}}} = 1 - 0.33 \left( \frac{p_w}{p_r} \right) - 1.05 \left( \frac{p_w}{p_r} \right)^2 - 0.037 \left( \frac{p_w}{p_r} \right)^3
\]

\[
+ 0.89 \left( \frac{p_w}{p_r} \right)^4 - 0.46 \left( \frac{p_w}{p_r} \right)^5
\]

(11)

\[
\frac{N_p}{N_e} = 8\%:
\]

- Oil:

\[
\frac{q_o}{q_{o,\text{avg}}} = 1 - 0.111 \left( \frac{p_w}{p_r} \right) - 2.39 \left( \frac{p_w}{p_r} \right)^2 - 2.79 \left( \frac{p_w}{p_r} \right)^3 - 1.07 \left( \frac{p_w}{p_r} \right)^4
\]

(12)

- Water:
\[ \frac{q}{q_*} = 1 - 0.24 \left( \frac{p_r}{p} \right) - 1604 \left( \frac{p_r}{p} \right)^2 + 1212 \left( \frac{p_r}{p} \right)^3 - 0.367 \left( \frac{p_r}{p} \right)^4 \]  

(13)

Figures 7 and 8 compare analytical curves for each phase and at both stages of depletion.

**PRODUCTION OPTIMIZATION**

Production optimization is one of the possible applications of IPR curves. A simulator can be used instead of a IPR curve but with a much higher computation time. The procedure is obtained by coupling the reservoir, well and surface facilities and performing a nodal analysis type of calculation to obtain the production rates, where the inflow pressure is

\[ \bar{p}_e - \Delta p_{\text{res}} = p_{\text{wf}} \]  

(reservoir system)  

(14)

and the outflow pressure is given by:

\[ p_{\text{sep}} + \Delta p_{\text{TH}} + \Delta p_{\text{ch}} + \Delta p_{\text{TV}} = p_{\text{wf}} \]  

(production system)  

(15)

where the pressure drop in the reservoir, surface facilities, choke and wellbore are represented by the subscripts (res), (TH), (ch) and (TV).

The calculation of the inflow pressure is made with the equation of the analytical IPR and the calculation of outflow pressure uses multiphase flow correlations\(^{4,5}\) (horizontal, vertical, and multiphase flow through chokes\(^{6}\)), resulting in an iterative procedure, which converges when:

\[ p_{\text{wf}}^{\text{inf low}} = p_{\text{wf}}^{\text{outflow}} \]  

(16)

When Eq. 16 is satisfied the equilibrium flow rate is obtained. With the production rate for each timestep the present value (PV) of the cumulative oil production is calculated by inputting the oil price and an interest rate.

Only commercial (discrete) tubing diameters and choke sizes are used in the optimization routine. This assumption simplifies the procedure and direct search can be used with a combination of the parameters values.

The results obtained for the two cases studied here are:

**Case 1**

Figure i shows the performance of PV as function of the time production for three diameters considered where it is observed that the maximum PV is obtained with the diameter of 2.375 in.

Figure j compares the results obtained by Schiozer\(^{10}\), using a reservoir simulator, with results from this work, using analytical IPR. It is observed that: (1) the maximum error is 5% and (2) in all cases studied, the same diameter and choke size are chosen using both procedures.

**Case 2**

For the calculation of equilibrium flow rates, total coefficients are obtained from analytical coefficients of each phase through analytical relations which are shown in Appendix A. With these coefficients total flow rate and then phase flow rates are calculated for each timestep.

**RESULTS - COMMENTS**

In the application of the IPR curves it is very important to consider the following aspects:

- the analytical IPR curves are dynamic curves which change with depletion. Therefore there is a different IPR curve for each stage of depletion;
- if one wants a IPR curve for a given time, an iterative procedure is shown in Appendix B to calculate the depletion stage and then the correct IPR curve, and
- more important that the coefficients obtained analytically, it is the average pressure of reservoir for each stage of depletion.

The last comment is justified by the fact that IPR curves have two fixed points (extremes) and if the coefficients vary, the impact in the curve shape is not very significant. However, figure m shows that the depletion stage is very important to calculate the production flow rate and, therefore, dynamic IPR curves are necessary.

**CONCLUSIONS**

The results obtained in this work yield the following conclusions:

1. an analytical procedure of calculation was developed to determine dynamic analytical IPR curves for limited and homogeneous solution-gas drive reservoirs, considering two and three phase flow, from results obtained from a reservoir Black-Oil simulator;
2. analytical IPR curves can be developed for any stage of depletion if the relative permeabilities and fluid PVT properties are known at different stages of depletion of reservoirs;
3. for two-phase flow, the mobility function must cover an adequate range of the adimensional variable \( \Pi \) (at least 70%) and for three-phase flow this range must be at least 90%; otherwise wrong analytical IPR curves can be obtained which, of course, are not adequate to represent the reservoir performance, especially for small times;
4. the results obtained with this procedure is compared satisfactorily with the results obtained using simultaneous simulation of reservoir and production facilities which requires much larger computational time.
5. it is shown that a simple automatic procedure can be developed to optimize production and, because only discrete values of each parameter are considered, this procedure can be used to optimize several parameters simultaneously.

**Nomenclature**

- \( B_g \) oil formation volume factor, (BO/STBO)
- \( B_w \) water formation volume factor, (BW/STBW)
- \( C_j \) adimensional coefficient, (fraction)
- \( D \) adimensional coefficient, (fraction)
- \( h \) height, (ft)
- \( k \) absolute permeability, (mD)


$k_j$  
N  
$p$  
$p_{cf}$  
$PV$  
$q_o$  
$q_{o,max}$  
$q_w$  
$q_{w,max}$  
$R$  
$t$  
$\mu$  
$\mu_w$  
$\phi$  

**Acknowledgments**

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**References**


**Table A: Data Summary, Case 1**

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<thead>
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<th>value</th>
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<td>absolute permeability</td>
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<td>critical gas saturation</td>
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<td>residual oil saturation</td>
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<td>drainage radius</td>
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**Table B: Data Summary, Case 2**

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<td>residual oil saturation</td>
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</tr>
<tr>
<td>drainage radius</td>
<td>900 ft</td>
</tr>
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</table>

**production system**

**horizontal section**

| separator pressure             | 40 psi |
| separator temperature          | 80 °F  |
| flowline length                | 5000 ft |
| inclination                    | 0.0 Degrees |
| diameter                       | 0.0167 ft |
| roughness tube                 | $5.0 \times 10^{-4}$ ft |
| vertical section               |       |
| wellhead temperature           | 70 °F  |
| depth                          | 5325 ft |
| inclination                    | 90 Degrees |
| roughness tube                 | $1.5 \times 10^{-4}$ ft |
Figure A: Influence of depletion on Oil Mobility Function

Figure B: Influence of oil production flow rate on Oil Mobility Function

Figure C Model Application

Black-Oil reservoir simulator

Calculation of phase mobilities and pressure variations

Calculation of IPR curve coefficients

IPR application (ex: coupling reservoir, wells, and surface facilities)

Figure D: Oil Mobility Function, Case 1

Figure E: Comparison between two different stages of depletion, Case 1

Figure F: Mobility function profile at 1% of depletion, Case 2
Figure G: Comparison of analytical oil IPR at two different stages of depletion, Case 2

Figure H: Comparison of analytical water IPR at two different stages of depletion, Case 2

Figure I: Present Value of oil cumulative production as a function of the time for three different tubing diameters, Case 1.

Figure J: Comparison between results from optimization procedure using simulation and IPR curves, Case 1

Figure K: Present Value of oil cumulative production as a function of the time for three different tubing diameters, Case 2.
Figure L: Results obtained with the analytical IPR for 3-phase flow, Case 2

Figure M: Dimensional IPR curves at different stages of depletion.

APPENDIX A
TOTAL COEFFICIENTS IN MULTIPHASE FLOW

Starting with analytical IPR for oil and water phases, and making

\[ x = \frac{p_{wf}}{p_e} \]  

(A-1)
equations become

\[ q_o = (1 + C_{1O}x + C_{2O}x^2 + C_{3O}x^3 + C_{4O}x^4)q_{o,\max} \]  

(A-2)
\[ q_w = (1 + C_{1W}x + C_{2W}x^2 + C_{3W}x^3 + C_{4W}x^4)q_{w,\max} \]  

(A-3)

Total rate flow is:

\[ q_t = q_o + q_w \]  

(A-4)
Substituting Eqs. (A-2) and (A-3) in (A-4), we have

\[ q_t = (1 + C_{1O}x + C_{2O}x^2 + C_{3O}x^3 + C_{4O}x^4)q_{o,\max} \]
\[ + (1 + C_{1W}x + C_{2W}x^2 + C_{3W}x^3 + C_{4W}x^4)q_{w,\max} \]  

(A-5)
and considering that:

\[ q_{t,\max} = q_{o,\max} + q_{w,\max} \]  

(A-6)
we obtain:

\[ q_t = q_{t,\max} + \left( C_{1O}q_{o,\max} + C_{1W}q_{w,\max} \right) x \]
\[ + \left( C_{2O}q_{o,\max} + C_{2W}q_{w,\max} \right) x^2 \]
\[ + \left( C_{3O}q_{o,\max} + C_{3W}q_{w,\max} \right) x^3 \]
\[ + \left( C_{4O}q_{o,\max} + C_{4W}q_{w,\max} \right) x^4 \]  

(A-7)
Finally, dividing Eq. (A-9) by \( q_{t,\max} \) we get

\[ \frac{q_t}{q_{t,\max}} = 1 + C_{1T}x + C_{2T}x^2 + C_{3T}x^3 + C_{4T}x^4 \]  

(A-8)
where the total coefficients are:

\[ C_i = \frac{C_{iO}q_{o,\max} + C_{iW}q_{w,\max}}{q_{t,\max}} \]  

(A-9)
where \( i = 1,2,3,4 \).
APPENDIX B
PROCEDURE TO USE IPR AS DYNAMIC CURVES
(AS A FUNCTION OF DEPLETION)

1) Calculate analytical IPR at different stages of depletion
(Ex. 1%, 3%, 5%, 7%, 9%, ...).

2) With the IPR for 1% depletion, calculate the equilibrium
flow rate through nodal analysis.

3) Repeat the following procedure until time or depletion
wished.
   a) Calculate the timestep to arrive to the valid IPR (Ex: 0
to 2%, 2 to 4%, 4 to 6% and 8 to 10%, ...)
   \[ \Delta t_i = 0.02 \left( V_T \phi (1 - S_w) \right) / q_1 B_o \]  
      (B-1)

   b) Calculate final and average time for each depletion
interval (Figure 14):
   \[ t_m = t_i + \Delta t_i / 2 \]  
      (B-2)
   \[ t_f = t_i + \Delta t_i \]  
      (B-3)

   c) Calculate the PV of the oil production in this timestep
using:
   \[ PV_i = \frac{q_i \Delta t_i \rho}{(1 + R)^\gamma} \]  
      (B-4)

   d) Go to the next IPR (and timestep)
   e) The PV of the cumulative oil production is the
summation of all PV_i.

Other procedures can be used to perform the same task.
For example, if a constant timestep is necessary, the
deployment stage can be calculated after each timestep to
determine which IPR must be used.