DEVELOPMENT OF A 2D STREAMLINE SIMULATOR FOR INCOMPRESSIBLE FLUIDS WITH HORIZONTAL VARIATION ON OIL DENSITY

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Abstract

Streamline-based simulators can be faster than conventional finite difference simulators. However, while some conventional finite difference simulators have the “API Tracking” option to allow the modeling of horizontal oil surface density variation, current streamline simulators can only consider constant API density throughout the reservoir. In order to study the implementation of an option equivalent to “API Tracking”, a two-dimensional streamline simulator was constructed which can model incompressible water and incompressible oil with horizontal density variation. Two oil properties tables, defining two extreme end-point densities, are used. The properties of oil in each cell are interpolated between these two extremes. After the pressure calculation in each time step, the new saturation and oil density profile along the streamlines are calculated using a mass conservative formulation. Then, these properties are averaged in each cell and the pressure is recomputed for the next time step. The results of the simulator were compared with the results of a commercial simulator, showing the viability of the method.

1. Introduction

There are several offshore heavy oil fields where a horizontal density variation can be found. The observed variation can be explained by several reasons. Wenger (2002) showed that one possible reason was associated with petroleum biodegradation varying with reservoir temperature and water depth: "the timing of hydrocarbon charge(s) and the post-charge temperature history of the reservoir can have major effects on oil quality". In reservoirs that enclose

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large areas, this variation yields a significant effect in the definition of development plans. For instance, the productivity of wells and the transmissibility between injector and producer well are functions of oil properties.

To study this type of reservoir, conventional Black-Oil simulators with the "API Tracking" option have been used (Geoquest Reservoir Technologies, 2004; CMG, 2004). The “API Tracking” option allows the modeling of horizontal oil density variation at the start of the simulation and tracking of the oil gravity throughout the run. This approach is acceptable but, since large models must be constructed and several simulation runs are required in most studies, there is need to reduce the simulation time.

Streamline simulation is an accepted and fast technique to model large, heterogeneous reservoirs with water injection as a mechanism for reservoir pressure maintenance (Thiele, 2001). Streamline-based simulators can be faster than finite difference simulators. Other advantages of streamline simulators are reduction in grid-orientation effects and quantitative flow visualization (Datta-Gupta, 2000). However, these simulators still have few options for production/injection control. They also consider only constant oil gravity throughout the reservoir.

In this work a mass conservative formulation was developed in order to solve both the saturation of water and the oil gravity along the streamlines in a two-phase case. To test this formulation, a two-dimensional streamline simulator was constructed. The program developed here can model incompressible water and incompressible oil with horizontal density variation like the API Tracking option.

2. Simulation with API Tracking

There are at least two commercial simulators which have an option called “API Tracking”. This option permits someone to define different API values to each cell. The properties of the oil to be used inside a cell are determined by means of interpolation of a set of oil property tables. During the pressure solve the oil gravity is held constant. Then a mass conservation equation is solved to update the oil surface densities that will be used to solve pressure for the next time step.

There are two ways to do this kind of simulation. The first one consists in splitting the reservoir in several regions. Inside each region a different relation between API and depth is defined. The second way consists in defining API values, pressures, saturation and solubility to each cell of the model, according to the API distribution that is required. To do this, an external initialization is necessary.

API Tracking used in commercial finite difference simulators works relatively well. However in a large reservoir, excessive simulation time may be an issue.

3. Streamline Simulation

In streamline simulators the equations are arranged in such a way that pressure field is initially solved at a specific time step independently of the saturation solution. After that, the saturation field is solved using smaller time steps (Batycky et al., 1997). To determine the new saturation field, streamlines are traced throughout the reservoir using the Pollock (1998) method and their saturation profiles are solved. Then the new saturation profiles in the streamlines are used to map new averaged saturations at each cell and a new time step is started (Figure 1).

![Figure 1 – Schematic of the streamline simulation scheme.](image)

3.1. Tracing Streamlines

Three-dimensional streamlines are traced according to the Pollock (1998) method. Knowing the velocity of the fluid at each cell boundary and the point where one line enters the cell, the exit point is determined using the calculation of the time that a neutral tracer particle would take to reach each of the outlet faces. In an incompressible case this
process always starts in a cell where there is an injector well and the streamline is traced until a producer well is reached.

3.2. Streamline Solution

Mass conservation of a phase \( j \) for a multiphase flow gives:

\[
q_j \rho_j - \nabla \cdot (\rho_j \vec{u}_j) = \frac{\partial (\rho_j S_j \rho_j)}{\partial t}
\]  

(1)

where \( q_j \) is the volumetric flow of phase \( j \) into the reservoir per unit of bulk rock volume. Applying the concept of phase fractional flow and considering incompressible system (\( \nabla \vec{u}_f = 0 \)) the mass conservation of the water phase becomes:

\[
q_t f_w - \vec{u}_t \nabla f_w = \phi \frac{\partial S_w}{\partial t}
\]  

(2)

where the subscript \( t \) means total of all phases. This equation when used in a streamline coordinate system becomes one-dimensional:

\[
q_t f_w - u_t \frac{\partial f_w}{\partial S} = \phi \frac{\partial S_w}{\partial t}
\]  

(3)

where \( u_t \) is the modulus of \( \vec{u}_t \).

During the process of tracing the streamlines, the time that the particle spends to cross the cells is added defining the time of flight (\( \tau \)) that is a key concept in streamline simulation. As \( \tau \) is determined, the saturations of the cells crossed by the streamline are picked up and an initial profile of saturation as a function of time of flight is determined.

Along a streamline,

\[
\tau = \int_0^\tau \frac{\phi}{u_t} d\varepsilon \quad \text{and} \quad d\tau = \frac{\phi}{u_t} ds
\]  

(4)

Applying (4) in (3) and considering that there aren’t sources along streamlines we find (Blunt et al., 1996):

\[
\frac{\partial f_w}{\partial \tau} + \frac{\partial S_w}{\partial t} = 0
\]  

(5)

where \( f_w \) is a function of \( S_w \). Streamline simulators solve this equation independently for each streamline. In simple cases, a dimensionless Buckley-Leverett solution can be used, with is very fast in comparison to a finite difference simulator (Batycky et al., 1997). In more complicated cases, where there are non-uniform initial conditions along a streamline, a numerical solution will be necessary. However, even in these cases there is a great reduction in simulation time since the same pressure solution can be used while the saturation profile moves through several grid blocks.

3.3. Gravity

It is also possible to consider the gravity in streamline simulations. According to Batycky et al. (1997) and Bratvedt et al. (1996), the problem can be solved splitting the equation in two parts and taking a temporary saturation distribution after the solution of (6), which is followed by the tracing and solution of vertical lines that consider the gravity part of the problem.

4. Solution for the Oil Density
Considering biphasic flow of oil and water, without gravity, Equation 1 can be used for both the water phase and the oil phase. For the water phase we can solve for saturation along streamlines as described in Section 3.2. For the oil phase we will have, after applying the concept of oil fractional flow:

\[ q_i f_{o} \rho_o - \bar{u}_i \nabla (\rho_o f_{o}) = \phi \frac{\partial (S_o \rho_o)}{\partial t} \]  

(6)

assuming incompressible rock, oil and water and non-uniform oil density.

Along a streamline we have:

\[ \phi \frac{\partial (S_o \rho_o)}{\partial t} + u_i \frac{\partial (f_{o} \rho_o)}{\partial s} = 0 \]  

(7)

Applying the same coordinate transform used in Section 3.2:

\[ \frac{\partial (S_o \rho_o)}{\partial t} + \frac{\partial (f_{o} \rho_o)}{\partial \tau} = 0 \]  

(8)

Once the saturation is obtained by solution of Equation 5, Equation 8 can be solved numerically for the product \( S_o \rho_o \) in a scheme identical to the one used there. After that, the new oil density can be obtained by:

\[ \rho_{o}^{n+1} = \frac{(S_o \rho_o)_p^{n+1}}{S_o^{n+1}} \]  

(9)

where \( S_o^{n+1} = 1 - S_w^{n+1} \) and \( n \) is the time step number.

5. Implementation

In order to test the idea of tracking oil gravity through a streamline simulation a two-dimensional simulator for incompressible systems has been constructed and the numerical solution of Equation 8 has been implemented. Capillary pressure was not considered. The program was constructed using the FORTRAN 95 language.

Two sets of oil properties, defining two extreme end-point densities, must be supplied. Each set consists on the following properties: API degree, bubble pressure, formation volume factor at bubble pressure, solubility ratio, viscosity at bubble pressure and ratio between variations of viscosity and pressure. The properties of oil in each cell are interpolated between these two extremes. Viscosity is a function of both: standard oil gravity and cell pressure. The viscosities of the extreme oils are determined at the cell pressure and then these viscosities are linearly interpolated based on the reservoir oil density of the cell.

At each time step the pressure is initially solved considering the total flow of fluids at reservoir conditions. So, its solution isn’t dependent on the saturation solution. Here, the saturations and the oil gravity from the previous time step are used to calculate the mobilities between cells in an explicit scheme.

Following the pressure solution, an initial set of streamlines are launched from the injector cells and traced until a producer cell is reached according to the Pollock’s method. The user can specify the total number of lines initially traced. The number of lines to be launched on each boundary of an injector cell is proportional to the total rate crossing that boundary. The initial profiles of water saturation and oil gravity at reservoir conditions along streamlines are picked up from the grid cells and normalized in constant intervals of time of flight. Then the new profiles of saturation and the oil gravity are calculated by discretization of Equations 5 and 8 respectively. After this an average is made in each cell crossed by any of the initial lines to calculate new cell-averaged values of saturation and oil gravity.

After the initial streamlines are traced and solved a new set of streamlines are traced and solved in order to cover the cells that haven’t been reached by any streamline yet. To do so a scan is made starting from the cell that has the maximum harmonic mean of distances to all wells of the reservoir in a direction to the cell that has the minimum mean, until all cells of the reservoir have been reached by at least one streamline. This criterion was established in order to increase the probability of picking up cells not yet reached by an initial line. Each cell scanned is checked if it was already reached by any initial streamline. If it hasn’t, a line is launched in a direction against the flow velocity until an injector cell is reached determining the initial position of the line. Then the part of the line that leaves from the scanned
cell and goes until a producer cell is determined and the whole line is solved using the same process of the initial lines. After that the averaged saturation and oil gravity of the cells reached by the complementary lines and that weren’t reached by the initial lines are calculated in a similar way.
6. Comparative Results

Two two-dimensional cases were run using both a commercial finite difference simulator and the streamline method in order to compare results obtained by two different simulation approaches. The first case corresponds to a hypothetical “step” case with a big discontinuity in the oil properties (Figure 2). The second case corresponds to a more realistic “gradual” case (Figure 3). Both cases use a 30 x 20 x 1 grid with cells of size 100 m x 100 m x 1 m, a constant permeability of 5000 mD and a constant porosity of 25%. Table 1 shows the extreme properties of oil that were used. The initial pressure was 28439 kPa and the minimum pressure allowed in the bottom of the producer wells was 27459 kPa. These conditions guarantee that all pressures of the model will always have pressure above the bubble pressure.

![Figure 2 – Map of oil gravity for the “Step” case (API Degrees).](image1)

![Figure 3 – Map of initial oil gravity for the “Gradual” case (API degrees).](image2)

![Table 1 – Extremes properties of the oil.](image3)

<table>
<thead>
<tr>
<th>API Degrees</th>
<th>P_{bubble} (kPa)</th>
<th>B_o (m^3/std m^3)</th>
<th>R_o (std m^3/std m^3)</th>
<th>\mu_o (cp)</th>
<th>d\mu_o/dP (cp/kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>27244.</td>
<td>1.35</td>
<td>100.</td>
<td>1.</td>
<td>3.57 \times 10^{-5}</td>
</tr>
<tr>
<td>18</td>
<td>26263.</td>
<td>1.15</td>
<td>70.</td>
<td>3.</td>
<td>2.04 \times 10^{-5}</td>
</tr>
</tbody>
</table>

6.1. The “Step” Case

In the “step” case, one water injector well is located in the region with the oil of better quality and a producer well is located in the other region. This case has also been run using the Eclipse™ simulator developed by Geoquest. Maps of oil gravity obtained with both simulators after 7670 days of production can be seen in Figure 4. It is clear the similarity between them. The oil production, water injection and percentile water-cut curves, obtained with both programs, are shown in Figure 5. The results show that we can model initial variation and tracking of oil composition by streamline simulation.

![Figure 4 – Comparison between our simulator and finite difference for the “step” case – API degrees.](image4)
6.2. The “Gradual” Case

In the “gradual” case there is a continuous variation of oil gravity from 20 API degrees at the Southern side to 28 API degrees to the Northern side. A pair of injector/producer wells is located in the lowest API region of the reservoir and another pair is located in the better area.

Figure 6 show the comparison between our simulator and grid-based simulation for oil gravity after 7670 days for the “gradual” case. We can see again a good agreement between the results. At Figure 7 one can see comparisons between oil production and water-cut curves for both producer wells. It’s interesting to point here the increase of oil production that occurs after 1500 days of production for the well located in the Southern side (worse oil) in both simulators. This oil rate increase is caused by oil of better quality arriving to this region.
7. Conclusions

Applying oil mass conservation to the streamtubes represented by the streamlines, an uncompressible approach was developed to account for oil density variations in the reservoir. This approach was incorporated into a 2D streamline simulator in order to test the applicability of the method. The results of two different cases were compared with the “API Tracking” option of a commercial finite difference simulator and a reasonable agreement between oil gravity maps and production curves was observed. A perfect agreement was not expected since the two methods are based in very different solution procedure, both involving simplifications.

The work that was developed here shows that incorporation of an option similar to the “API Tracking” in streamline simulator is possible with little modification to present codes.

8. Nomenclature

\[ q_j = \text{volumetric flow of phase } j \text{ into the reservoir per unit of buck rock volume} \]
\[ \rho_j = \text{density of phase } j \]
\[ u = \text{Darcy velocity} \]
\[ \phi = \text{porosity} \]
\[ S_j = \text{saturarion of phase } j \]
\[ t = \text{time} \]
\[ f = \text{phase fractional flow} \]
\[ s = \text{spatial distance coordinate along a streamline} \]
\[ \tau = \text{time is flight} \]
\[ Q_s = \text{total flow rate inside a streamline} \]
\[ G_j = \text{gravity component of phase } j \text{ flow} \]
\[ \bar{z} = \text{absolute permeability tensor} \]
\[ g = \text{gravitational acceleration constant} \]
\[ D = \text{depth} \]
\[ k_{ij} = \text{relative permeability of phase } j \]
\[ \mu_j = \text{viscosity of phase } j \]

9. References


