Emulation based Uncertainty Analysis: Method and Concepts

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Introduction
Reservoir simulation is essential in the development and management of oil reservoirs, as it is used to forecast the reservoir behaviour, which is vital in the decision-making process. However, the actual properties of the sub-surface are highly uncertain, and thus so are the appropriate choices of the reservoir model input parameters and the predictions based on these models. Therefore, to obtain a reliable production forecast, reservoir models consistent with the dynamic data available from field production are identified in a process known as data assimilation. However, it can be too expensive computationally to perform a full comprehensive uncertainty analysis. A successful method within this context is that of uncertainty reduction via Bayes linear emulation, which solves the speed problem and facilitates a detailed exploration of input parameters and a robust subsequent uncertainty analysis.

This work describes the iterative emulator-based Bayesian uncertainty analysis methodology and concepts used. See Ferreira et al. (2019) for further information.

Optimization versus Exploration
Many methods seek to optimise the match between reservoir simulator output and history data and hence produce one or a relatively small number of optimised reservoir simulator evaluations (Figure 1a). In many cases, this approach does not lead to adequate uncertainty analysis. In fact, a full uncertainty analysis demands a comprehensive exploration of the uncertain input parameter space of the reservoir model, to identify all input parameter configurations that would lead to acceptable matches between reservoir simulator output and history data (Figure 1b and Figure 1c).

\[ f_i(x) = \beta_i \cdot g_i(x) + u_i(x) + \delta_i(x) \]

(1)

Figure 1: 2D input space coloured by likelihood such that high values are desirable: (a) optimisation approach, (b) full analysis - a vast number of input configurations considered, (c) full analysis - acceptable matches using simulator, (d) two different subsets of the acceptable set (red/blue) (e) full analysis - acceptable matches using emulation.

Usually, a vast number of simulation runs to perform the uncertainty analysis is required, which becomes a critical issue. The challenge needs to be addressed considering the possible drawback of identifying only a partial subset of the input parameters consistent with observed data (red and blue points in Figure 1d). Critically, these subsets will result in unjustifiable forecasts that are both biased and overconfident, which directly lead to sub-optimal decision making. Figures 1d and 2 demonstrate this principle: the blue points and red points lie in different parts of the acceptable input space. Both are consistent with the history data, but the forecasts from the blue points or red points on their own are biased high/low while simultaneously being overconfident. This problem is even worse when the acceptable input space is composed of disconnected regions, in which case optimisers will often get stuck in one of these regions, again leading to biased, overconfident forecasts.

A full exploration of the input space is, therefore, critical. Hence, we construct emulators and employ iterative uncertainty analysis to reduce the reservoir uncertainty for given observed data carefully. The slight inaccuracy of the emulator causes the difference between the exploration final result using emulation (Figure 1d) and simulation (Figure 1c), that in turn is a computer model which is not a perfect representation of the real system. The iterative process used allows to obtain more detailed emulators at each iteration and reduce such difference.

Figure 2: Schematic production data from Figure 1d subsets (red lines - red points, blue lines – blue points, black error bars – historical data).

Emulation
An emulator is a statistical construct that seeks to mimic a complex physical model, such as the reservoir simulation model, but which is several orders of magnitude faster to evaluate. The emulator provides an understanding of the structure of the model’s behaviour and can replace the simulation model in many complex calculations. An emulator provides not only an estimate of the simulator model output at an unexplored input location but also an associated uncertainty statement regarding that estimate. The emulator is a vector function represented by where the vector \( x \) is the list of reservoir input parameters such as water-oil contact and the vector \( f(x) \) is the list of model outputs, such as fluid rates at different production times, with individual outputs denoted by \( f(x) \). The subset of the inputs \( x \) that are most influential for output \( f(x) \) is the vector of active inputs \( x_{\text{active}} \) for each output \( i \).

The first term on the right-hand side of eq. (1) expresses the global variation of \( f(x) \), where \( \beta_i \) are unknown scalar coefficients and \( g_i \) are known deterministic functions of \( x_i \), a common choice being low order polynomials. The term \( u_i(x) \) is a Gaussian process over \( x_{\text{active}} \) that expresses the residual local variation in \( f(x) \) not captured by the trend, and the nugget \( \delta_i(x) \) is an uncorrelated term that models the effects of inactive variables as white noise.
Figure 3 presents the schematic process to construct an emulator. A set of simulation runs is designed considering the uncertain inputs and then simulated using computer software. We use both inputs, and correspondent model outputs to construct the mathematical equation that relates them, which is then used as a substitute for the reservoir model depending on the objective of the project.

Figure 4 shows an example of a 1D emulator for the function, for which six model evaluations have been performed, given by the black dots. The emulator expectation evaluated over a large number of input points is given by the blue line, while the green lines give the emulator credible interval. We note that even after only six evaluations, the emulator expectation mimics the shape of the sine function well. Also, we see that the credible interval is large in-between model evaluations where the emulator is less informed, but shrinks to zero close to the evaluations as is desirable for a deterministic function.

Figure 5 shows an example of the first wave of an uncertainty reduction via Bayes linear emulation as applied to the toy example: (a) the six model evaluations in the first wave, (b)-(d) the three evaluations in wave 2, included sequentially.

Conclusion
Reservoir simulation is an essential tool for decision making, as it is used to forecast reservoir behaviour. Therefore, appropriate treatment of available historical data and reservoir uncertainties is necessary to provide reliable simulation models to be used in reservoir development and management. However, it can be too expensive computationally to perform a full comprehensive uncertainty analysis of the reservoir model. This work presented the description of the Uncertainty Reduction via Bayes Linear Emulation methodology, which is one way to solve the speed problem and enable a detailed exploration of input parameters and robust subsequent uncertainty analysis.

References

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